University of Chicago

A REDETERMINATION OF THE COEFFICIENT OF VISCOSITY OF AIR

A DISSERTATION

SUBMITTED TO THE FACULTY OF THE OGDEN SCHOOL OF GRADUATE SCIENCE IN CANDIDACY FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

(DEPARTMENT OF PHYSICS)

BY

ERTLE LESLIE HARRINGTON

A Private Edition
Distributed by
The University of Chicago Libraries

Reprinted from THE PHYSICAL REVIEW, N. S., Vol. VIII, No. 6, December, 1916

3

437

A REDETERMINATION OF THE ABSOLUTE VALUE OF THE COEFFICIENT OF VISCOSITY OF AIR.

[Reprinted from the PHYSICAL REVIEW, N.S., Vol. VIII, No. 6, December, 1916.]

A REDETERMINATION OF THE ABSOLUTE VALUE OF THE COEFFICIENT OF VISCOSITY OF AIR.

BY ERTLE LESLIE HARRINGTON.

IN view of certain results on the "Coefficients of Slip" obtained by him from his study of the laws of fall of small spheres in gases, Prof. Millikan suggested to me that I attempt to check these results by making some measurements on the coefficients of viscosity at very low pressures with the constant deflection apparatus designed by himself and Dr. Gilchrist. In the course of this work the method was so perfected as to make it capable of a precision apparently unapproached heretofore in measurements on the viscosity of gases.

Since the knowledge of the exact value of this coefficient for air is of such fundamental importance in many lines of physical research it seemed worth while to turn aside for the sake of attempting again to fix its value with all the precision possible. This was especially needed since some question[1] has been raised recently relative to the reliability of the estimate made by Prof. Millikan as to the most probable value of this constant. From determinations made under his direction by Gilchrist[2] by the constant deflection method, and by Rapp[3] by an improved capillary tube method, taken in connection with reliable determinations made elsewhere and by other methods, Prof. Millikan published[4] as the most probable value of this constant, η, at 23° C., .0001824, and estimated its uncertainty at not more than one tenth per cent.

In connection with another problem, Vogel,[5] in 1914, was led to make and publish a summary of the results of nearly all the determinations of this constant that had ever been made, but he arrived at a value about .8 per cent. higher than the above. In this, however, he includes determinations which certainly involve gross error and which are at least very far from what is now known to be the approximate value of η. In fact, he lists values with a total range of nearly 11 per cent., even in the same method, while observers with present laboratory methods and facilities,

[1] A. Gille, Ann. der Phys., 48, p. 799, 1915.

[2] PHYS. REV., I., p. 124, 1913.

[3] PHYS. REV., 2, p. 363, 1913.

[4] Ann. der Phys., 41, p. 759, 1913.

[5] Ann. der Phys., 43, p. 1235, 1914.

using any method whatsoever, would admit no greater error than a fraction of a per cent., hence the inclusion of such widely variant values of η renders more doubtful the validity of any mean so obtained.

The determination by Gilchrist appears also in this summary by Vogel, and although he gives it relatively more weight than any other single determination by any method, he nevertheless expresses the feeling that the method and theory had not as yet received full development. Since the publication of the Gilchrist article the method has been used by Timiriazeff,[1] but his experimental arrangements were such as to make it better suited to relative than to absolute determinations. It therefore seemed important to subject the method itself to as critical a study as possible while making the new determination of η.

Apparatus.

The apparatus is the same as that used by Gilchrist save for the suspension and for certain other features which it was necessary to modify in order to adapt it to running in vacuum. The diagram shows the general arrangement of the apparatus. The outer, O, of two concentric brass cylinders is made to rotate with constant velocity about the inner, I, which is suitably hung by an elastic suspension. The viscosity of the air produces a drag upon the inner cylinder causing it to be deflected from its position of rest to such an angle that the restoring couple of the suspension brought into play exactly counteracts the drag of the air, the angle of deflection being measured by the usual mirror telescope and scale method.

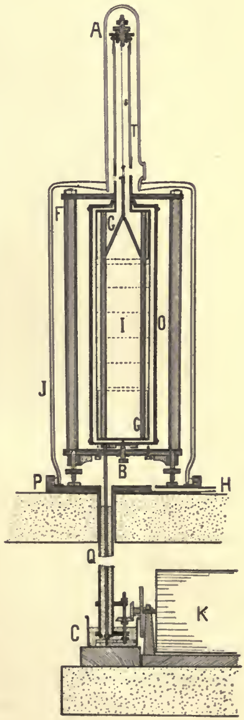

Fig. 1.

The cylinder frame, F, is mounted upon a heavy steel plate, P, accurately machined and having a raised rim in order to provide a mercury seal for the large glass jar, J, which covers the whole. To one side of the plate is drilled a hole to provide pump connections, and in the bottom is screwed a steel pipe, Q, enclosing the driving shaft, and of such length as to serve as a barometer column, and the lower ends of pipe, rod, and gearing were suitably constructed to dip into a mercury cup,

[1] Ann. der Phys., 40, p. 971, 1913.

C, through which the motion must be transmitted. The cylinders are carried by a heavy brass frame, F, provided with leveling screws and levels. The outer cylinder is supported on a slightly conical bearing, B, at the bottom, and at the top rotates on the tube, T, as an axle. The tube, T, at the same time supports the suspension head, A, and is rigidly held by the main frame. To eliminate end effects the inner cylinder is provided at either end with a guard cylinder, G, of the same radius, and rigidly held at a small distance from it and in perfect alignment with it by three brass posts which connect the two guard cylinders and suitable end disks. The upper part of this guard system and the suspension tube above mentioned are constructed as one piece so that the upper guard cylinder is held accurately centered with respect to the upper part of the rotating cylinder. The lower part rests on a conical bearing which is a part of, and accurately centered with respect to the bottom of the outer cylinder. The suspension head, A, is so constructed as to permit considerable translatory motion in any direction, as well as rotatory, thus making possible accurate adjustment. The above apparatus was constructed with great care and precision by Wm. Gaertner & Co., and to them much credit is therefore due for the success of the experiment. Such accuracy in construction made it possible to have the guard cylinders very close (.025 cm.) to the suspended cylinder, and thereby completely eliminate the need of end effect corrections.

A chronograph, K, provided with extra weights serves the double purpose of driving the apparatus, and also of leaving a permanent record of the speed of rotation. It was so arranged that the cylinder could be instantly thrown out of gear and the chronograph used independently of the other apparatus, this being done each time the period of vibration of the inner cylinder was determined.

A Beckmann thermometer, calibrated with a standard Baudin thermometer, and reading directly to .01 degree was hung beside the outer cylinder. A high-grade Centigrade thermometer graduated to .1 degree was hung beside it to serve as a check. The room itself was one of the constant temperature rooms of the Ryerson laboratory; a basement room, with no windows, lined on all sides, top and bottom, with a 5-in. layer of cork, and provided with heavy inner and outer doors. The room was efficiently heated by a new style electric heater (furnished by the Lee Electric Radiator Co.) used in connection with a sensitive thermostatic control, and the air was constantly stirred by a large fan. The temperature control was such that during any run the temperature variation recorded by the Beckmann thermometer was never more than a few hundredths of a degree, and often no change at all was detected.

The air in the apparatus was kept dry by an enclosed dish of phosphorus pentoxide.

Critical Study of Apparatus and Determination of Dimensions.

Inasmuch as one of the objects of the experiment was to study the possibilities of the method as well as to find the absolute value of η, considerable time was spent on this phase of the work, and w enever

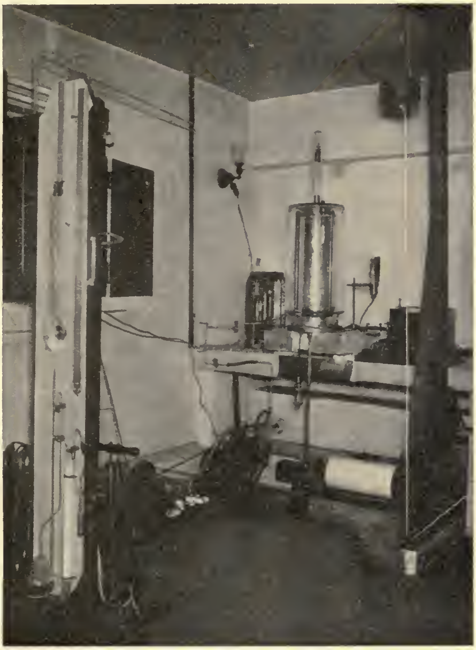

Fig. 2.

possible various methods of measurement were used for the purpose of cross checking.

1. *The Inner Cylinder.*—Three methods were used to measure the diameter of the inner cylinder. In order to insure getting different diameters and to entitle each different one to equal weight in the computation of the mean a large number of points evenly distributed over the surface of the cylinder were systematically numbered and the measurements were taken at these points. The first method was to place the cylinder vertically on the bed of a dividing engine having mounted at one

side with its axes perpendicular to the direction of motion a short focus telescope provided with cross hairs. The distance between tangent lines was then determined from the screw readings. The second method was to adjust the jaws of a large-size vernier caliper to the diameter of the cylinder and then determine the perpendicular distance between the locked jaws by placing the same on the bed of the dividing engine, and measuring by the usual method. The third method was to use a micrometer screw caliper of sufficiently large size to measure directly the diameter. The last two methods were by far the more convenient and yielded as accurate results. These two results differed by only 1 part in 13,000, and their mean agreed to Gilchrist's value to within 1 part in 6,000, which is a liberal estimate of the possible error in this dimension. The length was determined by cathetometer methods, and the value found to agree exactly with that given by Gilchrist, and is probably correct to within 1 part in 6,000.

2. *The Outer Cylinder.*—The most satisfactory method of measuring the diameter of the outer cylinder was found to be by filling it with distilled water observing the temperature and depth, and weighing the water used. Results thus obtained differed by only 1 part in 15,000, and the mean differed from that by Gilchrist by exactly the same amount, so the error is perhaps no more than 1 part in 8,000.

The accuracy in the inner surface was studied in this way: The inner cylinder, the lower guard ring, and the posts were removed, leaving the upper part of the guard cylinder system, which, as above described, form the axle for the outer cylinder. A heavy rod with one end hollowed out to fit the conical bearing in the bottom of the outer cylinder was inserted in the opening and brought to rest on the bearing. This rod carried a lever arm suitably curved to rest against the wall of the cylinder, capable of being adjusted to different heights, and bearing at the fulcrum a mirror. A suitable telescope with vertical scale placed at a distance of nearly three meters made it possible to detect the slightest deviation from constancy in the radius or perfection in symmetry. In doing this the lever was held lightly against the wall by a weight, and the cylinder slowly rotated at a constant speed. From the readings of the observer at the telescope and a measurement of all distances concerned, any variations could be quantitatively determined. It is important to note that this method not only tests the accuracy of the inner surface but tests also the symmetry of this surface about the same bearing that is to carry and hold in position the inner cylinder system. The results of this test were very satisfactory, inasmuch as the variations were of the order of 1 part in 2,500 and of such nature as to be self-counteracting in their

effect on the final results, which were therefore affected probably less than 1 part in 5,000.

3. *Moment of Inertia of the Inner Cylinder.*—This was found as usual by determining the period of vibration of the cylinder alone, and then when a known moment of inertia was added. The cylinder was suspended by piano wire and the cylindrical surface made plumb and symmetrical about the axis by suitable adjustment of the three support bars which connect the cylinder with the suspension clamp, and by use of small weights fastened to the upper supporting vane of the cylinder. The heavy, double support of the clamp from which the cylinder was suspended rested on a stone bench, and although the room was apparently free from air currents the cylinder was surrounded by a much larger one in order to insure entire absence of them. The passage through the zero position of the cylinder was indicated by the flash of a light into a telescope at the opposite side of the room, and an electric key enabled the operator to accurately register this time on the chronograph. The time divisions were marked by impulses from the standard laboratory clock and involved no appreciable error whatever, and the divisions could be read to one one hundredth of a second. By taking the period from a rather long run thus measured, the individual runs varied from the mean for any suspension by an amount of the order of 1 part in 10,000, which may be considered the probable error from this source. The added known inertia consisted of a bar and ring, accurately machined, and placed on the cylinder symmetrically. The dimensions of the ring and bar were determined on the dividing engine, and their weights by the analytical balance. The inertia of the combined ring and bar was computed by the usual formula derived for such bodies, and the result agreed to 1 part in 8,000 with that obtained by Gilchrist, this ratio probably representing the accuracy of this determination. No faulty adjustment to symmetry about the axis of suspension could have been constant to the different series of runs, since the ring and bar were frequently removed and replaced.

Having the two periods and the one known inertia, the moment of inertia of the inner cylinder was computed from the usual formula,

$$\frac{T_1^2}{T^2} = \frac{I + I_1}{I}.$$

The average departure of the various results from the mean for the moment of inertia of the inner cylinder obtained from the different suspensions used was 1 part in 5,000 and the mean differed from that obtained by Gilchrist by exactly the same amount.

DIMENSIONS.

Dimensions that are not involved in the computation of the results are here included for descriptive purposes, but only approximate values for such are given.

Vertical distance from base of chronograph to extreme top of cover	185 cm.
Diameter and height of glass jar, respectively	28 and 62 cm.
Length of outer cylinder	46
Length of suspension	23.5
Length of each guard cylinder	10
Length of inner cylinder	24.88
Distance between guard cylinders	24.93
Weight of inner cylinder	321 g.
Radius of outer cylinder	6.06317 cm.
Radius of inner cylinder	5.34116
Moment of inertia of inner cylinder (experimentally determined)	7,617.3

ADJUSTMENT OF INSTRUMENT.

Before being placed in the instrument the inner cylinder was again tested for symmetry and perpendicularity with the same methods and precautions used preliminary to the determination of its moment of inertia. In this three plumb bobs instead of one were used for the purpose of expediting matters and permitting simultaneous observations in three directions without in any way disturbing the cylinder or plumb bobs.

The lower end of the outer cylinder being in place, the inner cylinder and the guard cylinder system were put into position, fastened rigidly by the top brace, and after suspending the inner cylinder the suspension head screws and the leveling screws of the base of the instrument were so manipulated as to bring the surface of the guard cylinders into alignment with the inner cylinder as perfectly as possible without the use of a telescope. The outer cylinder was then put into position, the suspension tube again clamped, and the deflection of the inner cylinder noted as the outer cylinder was rotated at the speed to be used later in determinations. The suspension head was then rotated until the zero position was brought as much to one side as the deflection position was to the other side of the telescope which was in the middle of, and perpendicular to the scale. Three small support screws in the base of the instrument were then brought just to touch the bottom of the outer cylinder in order to hold it in its exact position during the final adjustment. The outer cylinder was loosened from its bottom and removed and the top brace replaced and fastened firmly in position. The final, and more careful adjustment of the guard cylinder system to alignment with the inner cylinder was made by means of a short focus telescope. With a suitable

clamp system attached to the frame the outer cylinder was replaced without at any stage of the process inclining or otherwise disturbing the inner cylinder. The preliminary adjustment to zero position made unnecessary the turning of either the guard cylinder system or the suspension head after this final adjustment, and thereby eliminated any error that might have come from a lack of perfect coincidence of the suspension with the axis of the suspension head collar. The use of the support screws for the base of the outer cylinder holds it, and therefore the lower end of the inner cylinder system, in precisely the position they occupy during a run. Keeping the guard cylinder system in place while replacing the outer cylinder eliminates the question as to whether it returns to the exact position it had during adjustment. With these precautions it would seem unlikely that any appreciable error could arise from faulty adjustment, and the results later given involving results obtained before and after dissembling and readjusting furnish convincing evidence of the absence of such error.

Trial Runs and a Study of Factors Affecting Results.

As will appear later, the value of η is computed from the time of rotation, the period of vibration of the inner cylinder, the deflection, the distance to the scale, and the temperature observations. The method of taking these observations was this: The zero position was read when the cylinder was at rest and checked by causing the cylinder to make small vibrations (about 1 cm. as seen on the scale) and determining the zero position as is done in using balances. This was done as a precaution against any error due to sticking, although experience showed this step really unnecessary. The outer cylinder was then put into rotation, the speed at first being modified by a brake attachment to the chronograph which could be operated from the position of the telescope in order to bring the inner cylinder quickly to near rest in its deflected position. As soon as a steady state was attained the stylus was lowered on to the waxed paper of the chronograph drum and the rotation continued until the stylus had traveled the length of the drum, which meant, with the speed used, an interval of about 13 min. In practice the inner cylinder was allowed to vibrate through two or three millimeters as seen on the scale, readings being taken at short intervals; this plan giving a large number of independent readings, eliminating the barely possible source of error due to sticking, and affecting a great saving of time that would otherwise be required to bring the cylinder to absolute rest, inasmuch as the period of vibration was very long, and the damping very small. At frequent intervals during this period the temperature was read on

the Beckmann thermometer. At the close of the run the cylinder was slowly let back and its zero position again checked. Now the outer cylinder was given a rotation sufficient to set the inner cylinder in vibration, and then thrown out of gear, thus allowing the chronograph to be used merely as such, whereupon the period of vibration was taken over a period of about 45 min. To do this an electric key at the position of the telescope enabled the operator to make accurate chronograph record of the passage through the zero position, the first five and the last five being recorded in order to provide means of cross checking and thus insure absence of appreciable error from this source. The periods thus obtained were probably accurate to within 1 part in 8,000.

The scale on which the deflections were read was a carefully selected straight meter stick tested with a standard metric steel scale. The magnification of the telescope was such that .1 mm. could be read easily and since the deflections were of the order of 600 mm. the readings were probably correct to within 1 part in 6,000. A steel tape was used to set the ends of the scale equidistant fiom the mirror and to determine its perpendicular distance from the mirror. The error in this was probably not greater than 1 part in 6,000.

Irregularities in the speed of the chronograph might offer a source of error, not on account of uncertainty as to what the speed is, for that is obtainable directly from the record, but on account of the resulting unsteadiness of the deflection. Fortunately it was found that the chronograph drove the apparatus at a very constant speed, though of course not perfectly so, owing perhaps to the irregularities in the friction, or a lack of perfection in the gearing. However, a small auxiliary weight, at the side of the operator, brought into series with the main weight by two pulleys, enabled him after some experience to almost completely neutralize any such irregularities, thereby reducing the error from this source to probably 1 part in 5,000.

The high consistency in the early trials in the results for any suspension indicated a satisfactory control in all the above sources of error, but it was found that variations in the suspension produced rather great variations in the results. In view of the experience of Gilchrist the bifilar form of suspension was tried at first, but on account of the rather great weight (321 g.) of the inner cylinder, and its small distance (.25 mm.) from the guard cylinders which permitted no sag, it was not possible to use silk fibers which permit, perhaps, the closest approach to the true bifilar type. Metallic ribbons were therefore used, but after two months of experience with them they were wholly discarded, since it was not found possible to entirely eliminate the effect of a change in the ribbon

or even of a mere change in the separation of the strands or in the manner of clamping, upon the results obtained. The trouble no doubt lies partially in the rather great discrepancy between such a bifilar and those ordinarily treated theoretically since the deflection brings into play not only a restoring couple due to the slight raise in the weight, but also a restoring couple due to the twist in the strands themselves. Such a suspension is therefore a sort of hybrid between the bifilar and the ordinary elastic unifilar suspension. The greatest error, however, probably comes from the fact that the ribbons have widths of the same order of magnitude as the separation of the strands which makes it quite likely that as deflection occurs the two edges of either strand may assume varying portions of the load, thereby causing a virtual change in the distance between the strands. If the strands be clamped at both ends it is unlikely that the load is equalized between the two strands, and if instead the strand be simply looped about a pin at one end in order to permit constant equalization of the load there is the possibility of error due to a rolling of the strands about the pin as the deflection occurs. Unrolled phosphor bronze wire in place of the ribbon was even less satisfactory owing to the residual coil and the consequent drift, nor was either found satisfactory later as a unifilar suspension. In fact the writer was led to conclude that little dependence could be placed on phosphor bronze where precise results are expected. Quartz fibers were tried, but it was not found possible to obtain fibers coarse enough to support the rather great weight and at the same time fine enough to give sufficient deflections. The smallest piano steel wire obtainable was much too stiff, but it was found possible by reducing the size of the smallest obtained, by very carefully rubbing with fine emery paper, to secure a sufficiently large deflection and yet retain adequate tensile strength. After the adoption of this plan no further suspension troubles were experienced. A number of such were made, and in all cases there was a good zero return and a satisfactory absence of drift. Moreover, different suspensions, though differing greatly in stiffness, yielded entirely concordant results. Later some samples of the uncoiled stock from which the hair springs of watches are made were furnished by the Elgin Watch Company and found to have the proper range of stiffness.

As will be seen later the torsion constant of the suspension is expressed in terms of the inertia of the inner cylinder and the period of vibration. From the observed damping, and the theoretical relation between the magnitude of damping and the effect on the period as given, for example, by Helmholtz, it was calculated that the total effect due to the damping factors would be of the order of 1 part in 10,000, or quite inappreciable.

Moreover, by experiment no effect on the period could be observed when the outer cylinder was removed and the guard cylinders were separated many times as far from the vibrating cylinder. But, although such factors as viscosity involved above do not appreciably affect the period, the fact must not be overlooked that the vibrating cylinder does carry the air with it, and the moment of inertia of this air must be taken into account. This point was first called to my attention by Dr. Lunn. The periods taken in vacua were actually found to be 1 part in 750 less than the periods taken at ordinary pressures, so the data given make due allowance for this effect.

Computation of Results.

For the calculation of the results the well-known and very simple formula[1] was used in this form:

$$\eta = \frac{\pi\phi I(b^2 - a^2)}{a^2 b^2 T^2 \omega l},$$

where η is the coefficient of viscosity, I the moment of inertia of the inner cylinder, a and b the radii of the inner and outer cylinders respectively, l the length of the inner cylinder, ϕ the angular deflection of the inner cylinder, T the period of vibration of the inner cylinder, and ω the constant angular velocity of the outer cylinder. If the period of rotation of the outer cylinder, t, be substituted for $2\pi/\omega$, and a constant, K, for the product $[I(b^2 - a^2)]/(2a^2b^2l)$ (having here a numerical value of 1.20188) the formula becomes:

$$\eta = \frac{tK\phi}{T^2} \quad \text{or} \quad \eta = \frac{tK}{T^2}\tan^{-1}\frac{s}{2d},$$

where s is the deflection as read on a straight scale, and d the distance of the scale from the mirror. The latter form was used in all computation. A development of the above formula involving a more general treatment which considers the coefficient of slip will be given in a paper, following this, which will consider the problem of viscosity at the low pressures where the effect of slip becomes appreciable. In apparatus of the dimensions here used the correction for slip at ordinary pressures amounts to about 2 parts in 100,000.

All determinations were made at temperatures in the neighborhood of 23° C., and the values for η reduced to that temperature by the use of the formula suggested by Prof. Millikan (l. c.),

$$\eta_{23} = \eta_\theta + .000000493(23 - \theta),$$

where η_θ is the value of the viscosity coefficient obtained at θ° C. This

[1] See Poynting and Thompson, Properties of Matter, p. 213.

simple formula, found satisfactory through the range mentioned by him must certainly hold in the small ranges here involved, which are, with but two exceptions, less than one degree.

Results.

The following data show the results of thirty-one determinations of η made at various times during about three months, and involve the use of six different suspensions. Suspensions C and D were really the same suspension under different physical conditions. In most cases during the series of runs for a given suspension the apparatus was taken apart and readjusted in order to make sure there was no error of adjustment.

Sus.	d (Cm.).	s.	θ.	t.	T.	$\eta_{23} \times 10^7$.
A	200.0	40.21	23.97	30.059	140.70	1,823.7
A	200.0	40.26	24.45	30.020	140.61	1,823.5
A	200.0	40.06	23.06	30.018	140.62	1,820.9
A	200.0	40.05	23.31	30.085	140.62	1,823.3
A	199.7	40.025	23.18	30.000	140.58	1,821.4
B	200.2	35.81	23.61	30.014	132.81	1,821.2
B	200.2	35.665	22.88	30.114	132.79	1,824.1
B	200.2	35.73	23.14	30.031	132.79	1,821.0
B	200.2	35.837	23.43	29.971	132.80	1,821.2
B	200.2	35.80	23.46	30.032	132.80	1,822.8
C	200.8	60.51	22.88	30.090	172.13	1,825.9
C	200.8	60.67	23.055	29.940	172.14	1,820.5
C	200.8	61.064	22.93	29.840	172.21	1,825.1
C	200.8	60.624	23.165	30.014	172.18	1,822.2
C	200.8	60.604	22.84	29.962	172.13	1,821.1
C	200.8	60.68	23.09	29.936	172.13	1,820.8
C	200.8	60.73	22.91	29.940	172.13	1,823.1
C	200.8	60.77	23.19	29.907	172.14	1,821.0
C	200.8	60.75	22.90	29.895	172.14	1,820.9
C	200.6	60.905	24.10	29.912	172.13	1,822.7
C	200.6	60.54	23.00	29.982	172.10	1,822.0
D	200.7	61.154	22.97	30.014	172.98	1,822.9
D	200.7	61.216	22.99	29.954	172.95	1,821.6
D	200.7	61.07	23.22	30.075	172.96	1,823.2
E	200.85	51.66	23.27	30.178	159.39	1,824.8
E	200.85	51.168	23.28	29.888	159.39	1,824.7
F	200.9	63.34	23.16	29.946	175.62	1,823.8
F	200.9	62.988	23.26	30.087	175.60	1,822.3
F	200.9	62.957	23.11	30.103	175.56	1,824.0
F	200.9	62.974	23.10	30.066	175.58	1,821.9
F	200.9	63.093	23.19	30.008	175.55	1,821.9

Mean value of η at 23° C. = 1822.6×10^{-7}.

Moreover, the runs show such variations in the various factors involved as to make each determination an independent one, and no determination made with satisfactory control in the manipulation of all steps was discarded.

The strongest evidence of the advantages of this method for the determination of η lies in the remarkable consistency of the results here shown. If from the individual deviations from the above mean one computes the probable error by the usual least square method the result is found to be .19 or 1 part in 9,600. Moreover, it should be emphasized that this really *includes every* source of error except those involved in the instrument constant, K. The probable error for each of the various determinations involved in this constant has been given above in detail and if the probable error in this constant be computed by the same method as above it is found to be 1.9 parts in 5,000. The combined or total error would be by this method of calculation only 1 part in 2,500 or .04 per cent. Moreover, if the means for the different suspensions be compared it is found that the maximum variation from the above mean is less than .03 per cent. with the one exception of suspension E, a watch-spring suspension the use of which was discontinued on account of its tendency to drift. Considering these two striking results, and making any reasonable allowance for any unreliability in the least square method of computing errors or of the estimates made of any individual probable error it seems entirely justifiable to claim that the above mean is correct to within less than .1 per cent. of the true value of η at the temperature considered.

A comparison of the consistency of these results with the lack of consistency of the results obtained by any other method, more especially by the capillary tube method, shows the marked superiority of this method. The uncertainties of the capillary tube method need not be considered here inasmuch as they are well known and have been discussed by Prof. Millikan,[1] by Fisher,[2] Vogel,[3] and others. Only a few points of contrast need be mentioned; the capillary tube method is based on incomplete theory since it does not consider the effect of the radial component of the velocity and other uncertainties due to the increase in the volume of the gas as it passes along the tube, the general end effects in addition to the question of the effect of irregularities in the bore upon the stream lines, the difficulties in getting the exact pressures, and above all, the impossibility of getting perfect tubes of the small radii usually employed, and the great difficulty of subjecting any tube selected to accurate examination as to uniformity, circularity, and even as to

[1] L. c. [2] PHYS. REV., 29, p. 147, 1909. [3] L. c.

the value of the radius itself, which enters, it should be recalled, in the fourth power. The wide variation in the results obtained by this method by different observers, and even by the same observer with different tubes, is ample proof that these uncertainties exist. On the other hand, with the method described above the theory is complete, there are no end corrections involved, no expansion takes place, every step in the construction of the cylinders is subject to control, the dimensions are so great that they may be determined with great accuracy, and every portion of the apparatus is subject to minute study for irregularities, and even should such exist, their effect would be far less serious than in the case of the other method. As to consistency with other observers we are essentially limited to the value obtained by Gilchrist who first used the method and made the claim that his result contained not more than .2 per cent. error. The value here obtained differs from his by less than that amount. That his values fluctuate through a greater range than the range here obtained is without doubt due principally to the suspension troubles mentioned above which were here so largely eliminated, for every determination of his of any instrument constant that could at this time be checked was most critically examined, and no one of them found to differ by more than 1 part in 6,000 from the value here given.

It is interesting to note that Rapp,[1] who perhaps came more nearly completely eliminating the errors in the capillary tube method than anyone else, and who used a large number of tubes, is practically identical with the result here obtained, as is also that obtained by Hogg,[2] who used an oscillation method. The result obtained by Grindley and Gibson,[3] using still a different plan differs only by about .03 per cent. More significant still is the fact that the value published by Prof. Millikan as correct to within .1 per cent. is less than .08 per cent. above the value here obtained, and as his value was based on perhaps the most accurate determinations by five different methods, it would seem that the above claim that the result here obtained is within .1 per cent. of the true value is well founded, since the above mentioned values all lie well within this limit.

In conclusion the writer wishes to acknowledge his gratitude to Prof. Millikan, who suggested the problem and maintained such constant and helpful interest in the research during its progress, and to Prof. Michelson, the head of the department, for various helpful suggestions.

Ryerson Physical Laboratory,
University of Chicago,
June, 1916.

[1] L. c.

[2] Am. Acad. Proc., 40, 18, p. 611, 1905.

[3] Proc. Roy. Soc., A, 80, p. 114, 1908.

[Reprinted from THE PHYSICAL REVIEW, Vol. 21, No. 3, March, 1923.]

A DETERMINATION BY THE CONSTANT DEFLECTION METHOD OF THE VALUE OF THE COEFFICIENT OF SLIP FOR ROUGH AND FOR SMOOTH SURFACES IN AIR.

BY LELAND JOHNSON STACY.

ABSTRACT.

Coefficient of slip for rough and smooth surfaces in air, determined by the constant deflection method.—The apparent coefficient of viscosity measured by this exceptionally precise method does not come out constant for the lower pressures unless correction is made for the slip at the surfaces. The relation $\eta_p'(1 + k\zeta_p) = \eta =$ constant, between η_p', the apparent viscosity coefficient, and ζ_p, the coefficient of slip, enables ζ_p to be determined from measurements of η_p' at pressures of 0.2 mm or lower. The apparatus included a vacuum-tight chamber inside which a cylinder of radius 5.341 cm was suspended by a steel wire concentric with a cylinder of radius 6.063 cm which could be rotated at a constant slow rate so as to cause a steady deflection of the inner cylinder. The accuracy of this method of measuring η_p' is so great that the values of $\zeta_{76} = \zeta_p p/76$ all lie within ± 4 per cent of the mean. The chief difficulty was in keeping the air pure because of the gradual evolution of gas, probably hydrogen, inside the apparatus; but by taking observations only shortly after evacuation this effect was avoided. *For brass surfaces*, ζ, reduced to 23° and 76 cm, came out 66.15×10^{-7} which is practically the theoretical minimum deduced by Millikan for a completely diffusing surface, 65.9×10^{-7}. *For surfaces coated with shellac*, the coefficient was found to be 97×10^{-7} for a fresh surface, in agreement with the value obtained by Lee from droplet measurements, but it decreased steadily with time, presumably because of a roughening due to oxidation, falling in two months to within 3 per cent of the theoretical minimum. The early part of this work was done in collaboration with E. L. Harrington.

Coefficient of viscosity of air at 0.1 mm is the same as at atmospheric pressure when correction is made for the slip effect. The constancy of the values obtained for ζ provides new evidence that the coefficient is independent of the pressure.

I. HISTORICAL DEVELOPMENT OF THE IDEA OF SLIP.

THE theory that the coefficient of viscosity of a gas should be independent of the pressure was first deduced by Maxwell[1] from a consideration of the internal friction of molecules assumed to be rigid spheres. He first deduced the relation

$$(1) \qquad \eta = \rho\bar{c}l,$$

where η = coefficient of viscosity; ρ = density of the gas; $\bar{c}$ = mean

[1] Phil. Mag., 1860, Vol. 19, p. 31.

molecular velocity; l = mean free path of gas molecule. Since the density is directly proportional to the pressure, while the mean free path is inversely proportional to the same quantity, the product ρl, and therefore η, should be independent of the pressure.[1] In a later paper,[2] Maxwell reported some experimental tests of his theory for pressures ranging from 30 in. to 0.5 in. of mercury. His apparatus consisted of a torsion pendulum of three plane-parallel plates suspended by an elastic fiber between four fixed plane-parallel plates. The whole system was enclosed in an airtight vessel and the logarithmic decrement of the oscillations was observed for different pressures. He found no observable change in the decrement, which is a measure of the viscosity, within the pressure range studied.

The same relation (1) was derived later by O. E. Meyer.[3] By an experimental arrangement similar to Maxwell's he checked the theoretical deductions for the same range of pressures. Results at pressures below 1/60 atmosphere showed a falling off of the viscosity coefficient, which he later ascribed to the fact that, in the theory of the experimental method, the external friction (ϵ) has been considered infinitely large in comparison with the internal friction or viscosity. Thus he introduced into the Kinetic Theory of Gases the slip coefficient $\zeta = \eta/\epsilon$, which Helmholtz[4] had previously defined for liquids.

II. Determinations of the Coefficient of Slip in Air.

The first experimental determination of the value of the coefficient of slip was made by Kundt and Warburg[5] who used the capillary tube method. Their results were:

Tube No.	Pressure.	Temperature.	ζ.	$\zeta_{76} = \frac{\zeta \cdot p}{76}$.
1	33.8 mm	15° C	.00017	76×10^{-7}
2	39.0 "	15° C	.00016	82×10^{-7}
2	33.8 "	15° C	.00018	80×10^{-7}
			Mean	79×10^{-7}

The mean of their values at 15° C and 76 cm is about 79×10^{-7}. Millikan in a preceding article has reduced this to 23° C, getting ζ_{76} at 23° C = 82×10^{-7}.

[1] Using Stokes' value $\sqrt{\eta/\rho} = .116$ for air, Maxwell made the first calculation of the mean free path of a gas molecule.

[2] Phil. Trans., 1866, Vol. 156, p. 249.

[3] Pogg. Ann., 1865, Vol. 125, p. 177.

[4] Wiener Sitzung., 1860, Vol. 40, p. 607.

[5] Pogg. Ann., 1876, Vol. 159, p. 399.

In the determination of the elementary electrical charge by the falling drop method Millikan[1] found that very small droplets of oil did not follow Stokes' equation

$$v = \frac{2}{9}\frac{ga^2}{\eta}(\sigma - \rho). \tag{2}$$

From his observations he made an empirical correction of the above equation, writing the corrected law in the form

$$v = \frac{2}{9}\frac{ga^2}{\eta}(\sigma - \rho)\left(1 + \frac{Al}{a}\right), \tag{3}$$

where Al was determined from the curve of his observations. Millikan pointed out that the correction of Stokes' Law for Slip[2] gave

$$v = \frac{2}{9}\frac{ga^2}{\eta}(\sigma - \rho)\left(\frac{1 + 3\zeta/a}{1 + 2\zeta/a}\right), \tag{4}$$

which, for small values of ζ/a, reduces to

$$v = \frac{2}{9}\frac{ga^2}{\eta}(\sigma - \rho)(1 + \zeta/a). \tag{5}$$

Thus, the Al determined by Millikan was really the coefficient of slip for oil and air. His value for 23° C was $\zeta_{76} = 77 \times 10^{-7}$. In a later determination[3] a more detailed study of the failure of Stokes' Law for small oil drops gave the value ζ_{76} at 23° C $= 82.2 \times 10^{-7}$.

For small drops of shellac, Lee[4] observed $Al = \zeta_{76}$ at 76 cm and 23° C to be 100×10^{-7}.

These experimental results led Millikan to a theoretical study of slip for different boundary conditions. He concluded that when no gas molecules were regularly reflected after impact upon the walls, the maximum of external friction, and hence the minimum of slip, would result. For a mechanically rough surface, which would cause such diffuse reflection of all gas molecules, he calculated the minimum value of ζ at 23° C and 76 cm to be 65.9×10^{-7}.

III. Theory for Slip Determinations by the Constant Deflection Method.

The theory of the Constant Deflection Method of determining viscosity coefficients gives

$$\eta = \frac{\pi I\phi}{\omega l T^2}\left(\frac{b^2 - a^2}{a^2 b^2}\right). \tag{6}$$

[1] Phys. Rev., 1911, 32, 382.

[2] Bassett, Hydrodynamics, Vol. II., p. 271.

[3] Phys. Rev., 1913, Vol. II., p. 139.

[4] Phys. Rev., 1914, IV., 420.

I = moment of inertia of suspended cylinder; ϕ = angular displacement of suspended cylinder; a = radius of suspended cylinder; l = length of suspended cylinder; T = period of oscillation of suspended cylinder; ω = angular velocity of outer cylinder; b = radius of outer cylinder, when it is assumed that there is no slip at the surfaces of the cylinders. At ordinary pressures, this assumption involves an error too small to be observed by this method. For the case where slip becomes appreciable the complete equation as developed by Millikan is

$$\eta = \frac{\pi I\phi}{\omega l T^2}\left(\frac{b^2 - a^2}{a^2 b^2}\right)\left(1 + 2\zeta_p \frac{b^3 + a^3}{ab^3 - a^3 b}\right). \tag{7}$$

The slip term at atmospheric pressure may be calculated for this apparatus since $a = 5.3412$ cm, $b = 6.0632$ cm, and $\zeta_{76} = 66 \times 10^{-7}$ (from theory). This gives a value for the term

$$2\zeta_p \frac{b^3 + a^3}{ab^3 - a^3 b} = .00002$$

which is quite negligible, since the experimental error is about .1 per cent. Hereafter the equation (6) will be written

$$\eta_p' = \frac{\pi I\phi}{\omega l T^2}\left(\frac{b^2 - a^2}{a^2 b^2}\right) \tag{8}$$

and η_p' defined as the apparent viscosity coefficient at the pressure p.

The value η_{76}' will be taken as the true value of the viscosity coefficient. Equation (7) may now be written

$$\eta = \eta_p'\left(1 + 2\zeta_p \frac{b^3 + a^3}{ab^3 - a^3 b}\right) \tag{9}$$

and solving for ζ_p

$$\zeta_p = \left(\frac{\eta}{\eta'_p} - 1\right)\frac{1}{k}; \qquad k = 2\left(\frac{b^3 + a^3}{ab^3 - a^3 b}\right). \tag{10}$$

Thus to determine the slip coefficient by this method it is necessary, first, to determine the coefficient of viscosity ($\eta = \eta_{76}'$) and then η_p' at some other pressure. Application of equation (10) gives ζ_p, and ζ_{76} is calculated from the formula $\zeta_{76} = \zeta_p(p/76)$. η' and η_p' must, of course, be observed at the same temperature or reduced to the same conditions by the proper formula.

The investigation of slip coefficients by this method was undertaken, at Prof. Millikan's suggestion, by E. L. Harrington [1] to determine whether the slip depended on the nature of the surface as the lack of

[1] Phys. Rev., 1916, VIII., p. 738.

agreement between the values for oil and shellac drops had indicated. Harrington devoted his time chiefly, however, to the improvement of the precision of the method of determining the viscosity coefficient, leaving the writer to carry on the slip determinations. The writer assisted Harrington for a short time in the first determinations of the slip coefficient. Some of Harrington's results will be included in this paper.

IV. Adaptation of the Apparatus for Slip Determinations.

The Constant Deflection Apparatus consists essentially of two concentric brass cylinders, the inner, I, being suspended on an elastic suspension, s, so that, when the outer cylinder, O, is driven at a constant speed by a clock driving mechanism, K, a constant torque due to viscosity will cause a constant deflection of the suspended cylinder from its equilibrium position. To eliminate end effects the inner cylinder is suspended between two guard rings, G, of the same diameter and less than .3 mm from it. A small mirror mounted at the base of the suspension wire makes it possible to observe the deflection by a telescope and scale. The period of rotation ($t = 2\pi/\omega$) of the outer cylinder was determined by a chronograph attached to the driving mechanism.

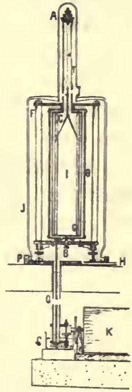

Fig. 1.

The constants of the apparatus as determined by Harrington were used.

Moment of inertia of inner cylinder	$I =$	7617.3
Radius of inner cylinder	$a =$	5.3410 cm
Length of inner cylinder	$l =$	24.88 cm
Radius of outer cylinder	$b =$	6.0632 cm

For work at low pressures, the cylinders were set upon a steel plate, P, about 30 cm in diameter. A large-mouthed glass bottle, J, about 28 cm in diameter and 62 cm high was found and the mouth ground plane to fit tightly a rubber gasket laid on the steel base-plate. An opening was ground in the bottom of the bottle for the suspension head, T, which projected some 25 cm above the large bottle. A glass tube, A, sealed at the top was fitted by a ground glass joint into the bottom of the large bottle thus closing the apparatus. A plate glass observing window was sealed over a small hole in the side of the suspension cover to prevent distortion of the image seen in the observing mirror. A discharge tube for spectroscopic work was also sealed into the side of the suspension cover. The driving shaft for the rotating cylinder was led through an iron pipe,

Q, sealed into the steel base-plate. The lower end of this pipe was immersed in a vessel of mercury, C, thus sealing it from the outside. A rim around the edge of the base-plate made it possible to use a mercury seal at this joint and, the bottom of the bottle being somewhat concave, a mercury seal was also used at the upper joint. Thus the apparatus was enclosed tightly enough to permit evacuation to about .001 mm pressure.

A Gaede mercury pump backed by a rotary oil pump made it possible to reduce the pressure within the apparatus to .1 mm in a little over two hours. The volume of the apparatus was about 150 liters. Pressures were read by a McLeod gauge calibrated to read .0001 mm. Temperatures within the apparatus were read from a Beckmann thermometer which had been calibrated by comparison with a standard Baudin instrument. The observing telescope and scale were mounted about 200 cm distant from the suspended mirror and the deflection could be read to .1 mm. The steel suspension wire used throughout this work gave an observed scale deflection of 62 to 64 cm when the period of rotation of the outer cylinder was about 30 sec.

V. Experimental Methods and Elimination of Errors.

The determination of the viscosity coefficient was made from the formula

$$\eta_p' = K\frac{t}{T^2}\tan^{-1}\frac{s}{2d} \qquad \text{where} \qquad t = \frac{2\pi}{\omega}; \qquad \phi = \tan^{-1}\frac{s}{2d},$$

$$K = I\left(\frac{b^2 - a^2}{2a^2b^2l}\right).$$

The period T and the scale distance d being determined beforehand, it was only necessary to observe s and t so that η_p' could be calculated. Temperatures and pressures were read as already described.

Having determined $\eta_{76}'(= \eta)$, it was necessary to determine η_p' at some low pressure and calculate ζ_{76} from the equations

$$\zeta_p = \left(\frac{\eta}{\eta_p'} - 1\right)\frac{1}{k} \qquad \text{and} \qquad \zeta_{76} = \frac{\zeta_p \cdot p}{76}.$$

For atmospheric pressure the observed scale deflection s was about 63 cm with an error of .3 mm. At 0.12 mm pressure and $t = 30$ sec. the scale deflection was found to be about 56 cm. Thus a difference of 7 cm with an error of .3 mm in reading would give an apparent error of not more than 1 part in 200. The gauge reading to .0001 mm, this error should be only 1 part in 1,200. Errors in observing all the other factors were much less than these, so they may be neglected. It was found,

however, that the principal source of inaccuracy was due to change of the pressure during observations. This rise of pressure was considerable in the first few hours after evacuation was stopped, but reached a steady value after 24 hours or so. It was found by experiment that during the 10-minute interval necessary for one complete observation the pressure change was from .001 to .003 mm. After one or two days the pressure rose less than that in 24 hours. This was explained by supposing the increase in pressure to be due to gases released from the glass and metal surfaces. By taking pressure readings before and after each observation a fairly accurate mean value was obtained. The temperature was also read at frequent intervals and the mean value used. The apparatus being set up in a constant temperature room, the observations were made within a very narrow range (22° to 24° C). Variations from 23° C were corrected for by Millikan's formula (Ann. der Physik, 1913, Vol. 41, p. 759).

$$\eta_\theta = \eta_{23^\circ} - .000000493(23^\circ - \theta).$$

In the early work on slip determinations, a series of observations was made after a single evacuation. The values of ζ_{76} calculated from these observations gave an initial value of about 70×10^{-7} but rose steadily until a value of 200×10^{-7} was found about two weeks later. The pressure change was from .1238 to .1448 mm during this interval. Since ζ_{76} should be independent of the pressure, this indicates that the increase in pressure must be due to the presence of some gas of a lower viscosity than air. This suggested that hydrogen (viscosity about one half that of air) was being released from occlusion by the metal parts of the apparatus. Spectroscopic examination of the discharge tube showed a definite increase in the intensity of the hydrogen lines when the apparatus was allowed to stand several days at a low pressure.

Admission of air to full atmospheric pressure to flush out the apparatus and a second evacuation gave the same result, viz., a low value during the first two or three hours after the pumps were stopped, then a steady rise in the value of the slip constant ζ_{76}. To eliminate this variation it was found advisable to admit air immediately after a set of observations was completed and to evacuate only a short time before readings were to be taken. Thus the time during which the "hydrogen effect" might be present was so short that it did not affect the results appreciably. It was found that the values of ζ_{76} obtained within three or four hours after an evacuation were quite consistent. Observations were usually made within an hour after evacuation. After many evacuations this "hydrogen effect" was less marked but was always present. By taking observations shortly after evacuation, it was avoided.

VI. Table of Observations and Calculated Data for Brass Surfaces in Air.

Results on Brass Surfaces in Air.

$d = 200.5$ cm; $T = 175.48$; $\eta = 1822.6 \times 10^{-7}$.

S (cm).	t (sec.).	θ (° C).	$\eta_p \times 10^7$.	$\zeta_p \times 10^5$.	p (mm).	$\zeta_{76} \times 10^7$.
56.28....	30.134	23.04	1643.0	3884	.1303	66.6
56.56....	30.040	23.05	1646.0	3811	.1328	66.6
56.71....	30.034	22.76	1649.4	3701	.1361	66.4
56.93....	29.989	22.81	1653.2	3616	.1388	66.0
57.23....	29.500	22.67	1634.7	4044	.1238	65.9
55.95....	30.267	22.67	1640.2	3912	.1266	65.2
56.76....	29.992	22.72	1646.0	3726	.1340	65.7
56.63....	30.124	22.72	1646.2	3721	.1362	66.7
55.77....	30.187	22.81	1628.2	4166	.1200	65.8
56.25....	29.975	22.82	1630.6	4107	.1209	65.3
57.25....	29.720	23.70	1645.0	3856	.1253	63.6
57.02....	29.880	23.66	1647.1	3797	.1268	63.4
57.10....	29.795	22.81	1644.8	3764	.1287	63.8
56.68....	30.032	22.81	1645.9	3738	.1311	64.5
56.71....	30.134	22.80	1652.9	3570	.1361	63.9
56.86....	30.079	22.83	1653.8	3556	.1367	64.0
57.20....	29.929	22.82	1655.1	3520	.1405	65.1
57.27....	29.922	22.79	1656.7	3480	.1428	65.4
57.15....	30.000	22.80	1657.6	3459	.1445	65.8
57.05....	30.053	22.73	1657.7	3451	.1459	66.2
57.32....	29.922	22.79	1656.2	3437	.1467	66.3
57.30....	29.930	22.77	1656.5	3444	.1475	66.8
56.51....	29.952	22.87	1640.5	3836	.1368	69.0
55.77....	29.917	22.84	1614.0	4469	.1178	69.3
55.65....	30.000	22.78	1614.6	4436	.1166	68.1
56.47....	29.873	22.90	1631.2	4053	.1300	69.3
56.53....	30.037	22.42	1641.9	3733	.1402	68.9
56.73....	30.043	23.01	1651.4	3405	.1508	67.6
56.62....	29.915	22.73	1637.0	3886	.1343	68.7
56.68....	30.102	22.84	1649.8	3591	.1349	63.7
56.46....	30.100	22.81	1647.1	3646	.1378	66.1
56.76....	30.018	22.79	1647.4	3630	.1424	68.0
56.68....	29.972	22.87	1642.6	3762	.1308	64.7
57.00....	29.860	22.99	1645.6	3710	.1338	65.3
57.03....	30.158	23.33	1662.9	3338	.1558	68.3
56.34....	30.080	23.46	1638.8	3915	.1258	64.8
Mean ...						66.15

The observations were made within a pressure range of .1 to .18 mm since at lower pressures the error due to change of pressure during an observation was large, while at pressures above .18 mm the deflection s differed so little from the deflection at atmospheric pressure that the error of observing this difference became considerable.

In general, the experimental conditions were as follows:

Temperature — 22° C to 24° C
Pressure — .1000 mm to .1800 mm
Period (t) — 30 sec. ($\pm .2$)
Period (T) — 175.5 sec.

$$\left.\begin{array}{lrl} \text{Scale deflection, for } p = & 76\ \text{cm} & - 630\ \text{mm} \\ \text{"} \quad p = & 0.18\ \text{mm} & - 580\ \text{mm} \\ \text{"} \quad p = & 0.10\ \text{mm} & - 550\ \text{mm} \end{array}\right\} \text{approximately}$$

Observations were usually made in pairs within an hour after evacuation.

The mean value (66.15×10^{-7}) is very close to Millikan's theoretical minimum value (65.9×10^{7}) but is considerably lower than any of the values found by other methods for oil or for glass surfaces.

VII. Determination of Slip for Shellac Surfaces in Air.

The cylinders were next coated with a thin layer of shellac, dried by an air blast and replaced in position. From the weight of the cylinders before and after the shellac was applied and the surface area of the cylinders, the average thickness of the shellac film was calculated. This was found to be .01 mm and made only a small correction in the values of the constants a and b.

After the determination of the viscosity coefficient at atmospheric pressure, the pressure was reduced and observations made as before. At first a high value of $\zeta_{76} = 97 \times 10^{-7}$ was found while the shellac was fresh. Later, the values of ζ_{76} dropped steadily toward the minimum value. This result is shown in the two sets of data here given; the first set of observations being due to Harrington and the second to the writer. In two other trials, accidental experimental difficulties prevented the writer from making observations while the shellac was fresh but values between 80 and 90×10^{-7} were found several days after application of the shellac. The fall in the observed value of the slip coefficient indicates a change in the surface due probably to oxidation of the shellac which seems to produce a rough surface.

Data on Fresh Shellac Surfaces in Air.

$d = 200.6$, $T = 176.00$.

E. L. Harrington: Fresh Shellac, August 1, 1916.

Date.	*S.*	*t.*	θ.	$\eta_p' \times 10^7$.	$\zeta_p \times 10^5$.	p (mm).	$\zeta_{76} \times 10^7$.
Aug. 9........	55.93	29.719	23.38	1602.8	4911	.1474	95.2
	55.72	29.727	23.44	1597.3	5059	.1478	98.5
Aug. 10, A.M..	53.42	29.715	23.76	1531.3	6842	.1048	94.3
	52.55	29.960	23.86	1519.4	7188	.1058	100.0
Aug. 10, P.M..	53.99	29.936	23.67	1559.0	6080	.1125	90.0
	53.19	30.263	23.60	1552.9	6240	.1133	93.0
Aug. 11, A.M..	53.43	30.227	23.46	1556.6	6121	.1107	89.0
	52.96	30.440	23.50	1553.9	6204	.1112	90.8
Aug. 11, P.M..	55.19	30.100	23.13	1602.0	4886	.1288	82.8
	55.14	30.100	23.13	1600.5	4920	.1298	84.1
Aug. 12.......	56.94	29.827	23.64	1637.0	4099	.1438	77.6
	56.75	29.917	23.70	1636.5	4108	.1438	77.7
Aug. 30.......	57.28	29.780	23.10	1653.1	3538	.1506	70.1

L. J. Stacy: Fresh Shellac, March 7, 1917.

Mar. 9, A.M..	56.04	29.789	23.36	1578.8	5588	.1330	97.7
	55.92	30.010	23.40	1582.5	5381	.1352	95.7
Mar. 9, P.M..	57.66	29.800	23.14	1619.7	4580	.1470	88.6
	57.26	30.080	23.19	1623.7	4486	.1482	87.5
Mar. 10, A.M..	58.15	29.723	23.55	1628.2	4414	.1418	82.3
	57.76	29.983	23.59	1632.4	4318	.1441	81.9
Mar. 10, P.M..	57.56	30.164	23.53	1636.7	4206	.1493	82.6
	58.12	29.896	23.50	1637.7	4179	.1543	84.8
	57.30	30.322	23.24	1637.9	4152	.1545	84.4
Mar. 11.......	56.83	29.985	22.87	1606.6	4876	.1313	84.2
	56.06	30.432	22.86	1608.7	4820	.1323	83.9
Mar. 12.......	57.38	30.334	23.12	1640.8	4066	.1487	79.5
	57.77	30.188	23.21	1643.8	3999	.1495	78.7
Mar. 13.......	57.75	30.144	23.52	1640.9	4102	.1443	77.9
	58.05	30.031	23.52	1643.1	4048	.1473	78.4
Mar. 16.......	58.16	30.169	23.07	1653.7	3747	.1518	74.8
Mar. 17.......	57.42	30.084	22.85	1628.4	4242	.1332	74.3

The following results were obtained using the same experimental method as with brass surfaces. The shellac surfaces were about two months old when the first readings were taken.

The results of these experiments furnish independent evidence of the fact that the viscosity of a gas is independent of the pressure, since ζ_{76} turns out to be a constant for a given surface in air. The value of ζ_{76} for rough surfaces checks Millikan's theoretically deduced values within the limit of error of the experiment. The variation in slip for different surfaces has been checked and the result for fresh shellac, $\zeta_{76} = 96.8 \times 10^{-7}$, is very close to the result Lee obtained, $\zeta_{76} = 100 \times 10^{-7}$, from the correction of Stokes' Law for falling shellac drops.

In conclusion the writer wishes to express his indebtedness to Professor R. A. Millikan who suggested the problem and directed the experimental

Data for Old Shellac Surfaces in Air.
$d = 200.6$ cm; $T = 176.00$ sec.

$S.$	$t.$	$\theta.$	$\eta_p' \times 10^7.$	$\zeta_p \times 10$	p (mm).	$\zeta_{76} \times 10^7.$
55.39....	30.027	23.67	1605.1	4833	.1088	69.2
57.49....	29.767	24.28	1648.5	3875	.1338	68.2
57.06....	30.076	24.34	1653.3	3767	.1370	67.9
57.49....	30.023	23.11	1661.5	3443	.1535	69.5
57.02....	30.058	22.76	1653.8	3639	.1444	69.1
57.20....	30.123	22.87	1662.5	3431	.1478	66.7
57.38....	30.023	22.90	1661.7	3447	.1488	67.5
57.28....	30.074	22.95	1662.1	3444	.1493	67.7
57.64....	29.885	22.92	1659.2	3507	.1478	68.2
57.72....	29.930	22.97	1664.4	3392	.1495	66.7
57.44....	30.144	23.08	1668.9	3299	.1573	68.3
57.60....	30.050	23.11	1668.2	3317	.1585	69.2
56.75....	30.111	23.24	1647.3	3830	.1351	68.1
56.90....	30.112	23.28	1651.1	3744	.1373	67.6
57.13....	30.067	23.21	1656.0	3613	.1437	68.3
55.19....	30.014	23.06	1597.2	5032	.1057	70.0
55.23....	30.087	23.10	1602.3	4910	.1076	69.5
57.15....	30.210	23.57	1664.0	3464	.1498	68.3
57.27....	30.271	23.66	1670.8	3316	.1533	66.9
56.91....	30.211	23.21	1657.2	3585	.1411	66.6
57.17....	30.174	23.31	1662.6	3469	.1445	66.0
57.36....	29.963	22.55	1650.5	3823	.1353	68.1
57.37....	30.019	22.58	1652.8	3793	.1380	68.6
56.74....	30.300	22.51	1644.5	3780	.1288	64.1
57.33....	30.161	22.87	1655.3	3742	.1356	66.8
57.25....	30.160	22.86	1652.6	3806	.1364	68.3
58.38....	29.508	21.94	1647.3	3835	.1363	68.8
57.23....	30.152	22.12	1655.5	3777	.1373	68.2
Mean ...						67.7

This result is $2\frac{1}{2}$ per cent higher than the value found for brass surfaces in air.

work; to the other members of the Physics Department of the University of Chicago for their interest and assistance throughout the investigation; and, in particular, to Dr. E. L. Harrington with whom the author worked in the first determination of slip by this method.

Ryerson Physical Laboratory,
University of Chicago,
September 22, 1922.[1]

[1] The work described in this paper was completed in February 1919.

Phys.Sci.
QC189 Harrington, E.
H37
A redetermination of the coefficient of viscosity of air.

(*Reprinted from* NATURE, *Vol.* 136, *page* 682, *October* 26, 1935.)

Viscosity of Air and the Electronic Charge

THE greatest uncertainty in determining the electronic charge e by the oil drop method of Millikan is introduced by the uncertainty in the assumed value of the coefficient of viscosity of air, η. The value adopted by Millikan in 1917

$$\eta_{23^\circ} = (1822 \cdot 6 \pm 1 \cdot 2) \times 10^{-7}$$

is probably too low, and its accuracy overestimated, as is pointed out by Shiba[1].

Considering the fundamental importance of the constant e, I have undertaken a new determination of η, using the rotating cylinder method also employed by Millikan and his co-workers[2,3]: An inner cylinder of electron metal, suspended vertically by a fine phosphor-bronze wire between two guard cylinders of equal diameter is deviated from its equilibrium position through an angle φ by a concentric outer cylinder, rotating with constant velocity, η being calculated from the equation

$$\eta = \frac{I\ (b^2 - a^2).\varphi\ t}{2\ a^2\ b^2\ l\ \ T^2}, \text{ where}$$

a = the radius of the inner cylinder = 2·81767 cm. at 20°;
b = the radius of the outer cylinder = 3·26628 cm. at 20° or = 3·18328 cm. at 20° (two different cylinders);
l = the length of the inner cylinder = 9·9981 cm. at 20°;
t = the time of revolution of the outer cylinder (20 – 150 sec.);
T = the period of oscillation of the suspended system (53 – 128 sec., using different suspensions);
I = the moment of inertia of the suspended system about the line of suspension = 423·22 gm.cm^2.

The mean value of 51 determinations of η for dry air at temperatures between 18·9° and 20·9° is

$\eta_{20^\circ} = (1820 \cdot 0 \pm 3 \cdot 0) \times 10^{-7}$ corresponding to
$\eta_{23^\circ} = (1834 \cdot 8 \pm 3 \cdot 0) \times 10^{-7}$.

From this we get

$$e = \left(\frac{1834 \cdot 8}{1822 \cdot 6}\right)^{3/2} \times 4 \cdot 770 \times 10^{-10} = (4 \cdot 818 \pm 0 \cdot 012) \times 10^{-10} \text{ E.S.U.},$$

the uncertainty stated being due only to the viscosity, other sources of error not being considered here.

I am, therefore, of the opinion that the discrepancy